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COMMON MISREPRESENTATION OF THE EINSTEIN– PODOLSKY–ROSEN ARGUMENT

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A frequently given version of the argument of Einstein, Podolsky and Rosen against the completeness of the quantum mechanical description is criticized as a misrepresentation that lacks the cogency of the original EPR argument.

Key words: Einstein–Podolsky–Rosen argument, quantum measurements, physical reality, completeness of quantum mechanics.

1. INTRODUCTION

The argument of Einstein, Podolsky and Rosen (EPR), advanced by these authors in support of the thesis that quantum mechanics cannot provide a complete description of a physical system, has been receiving close attention in foundational studies and debates on quantum mechanics ever since the famous paper of EPR appeared in 1935 [1]. Not lacking in subtlety, the EPR argument was bound to be misrepresented frequently in the extensive writings on the subject. A particular version of the EPR argument, analyzed critically as a misrepresentation already more than 25 years ago by Hooker [2], who classified it as “the third related EPR argument,” has been appearing persistently in the literature, especially in that aimed at a more general audience,¹ and in popular accounts.² The present paper is occasioned by a clearly stated occurrence of this version in a book on the relatively recent but already influential “consistent-histories,” or

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¹For example, even in the admirable analysis and interpretation of the philosophy of N. Bohr by Folse [3].

²See, for example, Ref. [4].

“logical,” revision [5–7] of the Copenhagen interpretation of quantum mechanics, written by one of its originators R. Omnès [8].

The important difference between the original EPR argument and, to use Hooker’s classification, “the third related” version will be pointed out using the presentation of Omnès; moreover, it will be argued anew that “the third related” version is a misrepresentation that loses the cogency of the argument as it was put forward originally.³ This will agree with the findings of Hooker [2], but for different reasons than those he advanced; Hooker’s reasons will be criticized as flawed on the physical grounds.

2. THE ORIGINAL EPR ARGUMENT

As Omnès says himself in his book, nothing can replace the direct reading of the original paper of Einstein, Podolsky and Rosen when studying their argument against the completeness of the quantum mechanical description. Following this advice should not be an onerous task as this famous paper has only four pages, in which the argument is developed clearly and succinctly—and the reader is entreated to peruse the paper to gain a first-hand knowledge of the matter under the present discussion. The following three paragraphs are a synopsis of the EPR argument, with comments that attempt to facilitate an appreciation of the intuitive physical context behind the formal logical structure of the argument.

A satisfactory theory should be complete, and a necessary condition for the completeness of a theory is that it has a counterpart for every “element of physical reality.” A sufficient criterion for recognizing an element of physical reality that corresponds to some physical quantity is satisfied when one can predict with certainty the value of that quantity without in any way disturbing the system to which the quantity pertains. Such a criterion aims at defining rigorously an empirically discernable symptom of the existence of physical reality that is independent of the observer and his measurements. For a *conserved* physical quantity, an intuitive justification for this criterion can be given easily along the following lines: if one can, *without in any way disturbing* the given system, predict the value of a conserved quantity pertaining to the system, the quantity must have had that value from the time of the last external disturbance of the system, and it will keep that value as long as the system remains undisturbed, in particular before any measurement is performed on the system, including a measurement of the quantity concerned. Generally, considering also nonconserving quantities, the EPR reality criterion is a condition that one would accept intuitively as guaranteeing, when it is fulfilled, that a measurement of the quantity concerned only *reveals* the quantity’s value, which exists independently of whether the measurement is performed or not.

³Of course, the EPR argument is compelling only when their concept of reality is accepted, see Secs. 2 and 3.

Having formulated their demands on a complete theory in terms of elements of physical reality that can be identified as such according to a well-defined criterion, EPR note that the wavefunction $\Psi(x_1, x_2)$ of a system consisting of two noninteracting subsystems that, however, interacted with each other in the past, in general can be expanded in terms of complete sets $u_n(x_1)$ and $v_s(x_1)$ of eigenfunctions corresponding to different physical quantities A and B , respectively, of system 1:

$$\Psi(x_1, x_2) = \sum_n \psi_n(x_2) u_n(x_1) = \sum_s \phi_s(x_2) v_s(x_1). \quad (1)$$

Here, the functions $\psi_n(x_2)$ and $\phi_s(x_2)$ can be regarded as the coefficients in the two expansions. The summations above run over at least two terms, and so the state (1) is now often called “entangled,”⁴ in the sense that it does not factorize into a single product of two functions where each depends only on the degrees of freedom of one of the subsystems. EPR point out that one can assign different wavefunctions to subsystem 2 by performing different local measurements on subsystem 1. This is because of what they term “the reduction of the wave packet” upon a measurement, now usually called the collapse of the wavefunction: when the quantity A is measured on subsystem 1 and found to have a value a_k , subsystem 1 is left in a state described by the wavefunction $u_k(x_1)$, collapsing the wavefunction (1) into a single term $\psi_k(x_2) u_k(x_1)$, which means that subsystem 2 is left in a state described (to within a constant) by a wavefunction $\psi_k(x_2)$; but if the quantity B , instead of A , had been chosen to be measured and a value b_r obtained, the state of the system would have collapsed to the state $\phi_r(x_2) v_r(x_1)$, leaving subsystem 2 in a different state $\phi_r(x_2)$.

The two wavefunctions ψ_k and ϕ_r may happen to be eigenfunctions of two noncommuting operators corresponding to conjugate physical quantities P and Q , respectively. EPR illustrate this by considering a two-particle system prepared in a state in which, along a given direction, the difference of the position coordinates of the particles and the sum of their momenta have sharp values x_0 and 0, respectively:⁵

$$\Psi(x_1, x_2) = \delta(x_2 - x_1 - x_0). \quad (2)$$

For simplicity, they assume only a one-dimensional motion, and use a non-normalizable delta function instead of a more realistic wave packet that would have a finite spread in position and momentum. The above entangled state can be written as

$$\Psi(x_1, x_2) = \int \psi_p(x_2) u_p(x_1) dp, \quad (3)$$

⁴This terminology goes back to Schrödinger [9].

⁵As observed immediately by Bohr in his reply to EPR [10], such a state is perfectly legitimate in quantum mechanics because the operators of the difference in the positions and of the sum of the momenta of two particles commute.

where

$$u_p(x_1) = \frac{1}{2\pi\hbar} \exp(ipx_1/\hbar) \quad \text{and} \quad \psi_p(x_2) = \exp[-ip(x_2 - x_0)/\hbar] \quad (4)$$

are momentum eigenfunctions of particles 1 and 2 with eigenvalues p and $-p$, respectively; it can be written also as

$$\Psi(x_1, x_2) = \int \phi_x(x_2) v_x(x_1) dx, \quad (5)$$

where

$$v_x(x_1) = \delta(x_1 - x) \quad \text{and} \quad \phi_x(x_2) = \delta(x_2 - x - x_0), \quad (6)$$

are position eigenfunctions of particles 1 and 2 with eigenvalues x and $x + x_0$, respectively. Now when a measurement of the momentum of particle 1 yields a value p , one knows with certainty that the momentum of particle 2 must be $P = -p$, because on that measurement the wavefunction (3) collapses to the state $\psi_p(x_2)u_p(x_1)$, where $\psi_p(x_2)$, the state in which particle 2 is left, is the eigenfunction in (4) of P with the eigenvalue $-p$. But one is free to measure the position of particle 1, instead of its momentum. If it was then decided to measure the former, on the resulting value x of such a measurement one could have predicted with certainty that the position of particle 2 is $Q = x_0 + x$, because the wavefunction (5) would have collapsed to the state $\phi_x(x_2)v_x(x_1)$, where $\phi_x(x_2)$, the state of particle 2, is the eigenfunction in (6) of Q with the eigenvalue $x_0 + x$. EPR assert that the momentum P and position Q of particle 2 must possess simultaneous reality according to their reality criterion, as the predictions of P and Q can be done by measurements on the noninteracting particle 1 and thus without disturbing particle 2 in any way.⁶ The formalism of quantum mechanics, however, does not allow a state in which a particle has simultaneously a definite momentum and a definite position, and so EPR conclude that the quantum-mechanical description is not complete. While EPR admit that one cannot predict the position and momentum of particle 2 simultaneously, they counter this objection by asserting that the reality of the position and momentum of particle 2 cannot, in any “reasonable definition of reality,” depend on the process of measurement on particle 1 carried out without disturbing particle 2. In other words, they argue that the simultaneous reality of the position and momentum of particle 2 is established already by the *possibility* of predicting either of the two quantities on the result of a measurement of *either* the position *or* momentum of the noninteracting particle 1. In drawing this conclusion, they use not only their reality criterion, but also rely on their firm conviction that in any “reasonable” concept of physical reality, the real state of a system must be independent of what may happen to another system, from which the system in question is well separated and with which it has no interaction.

⁶This can be ensured by having the particle separation x_0 sufficiently great; curiously, EPR do not mention explicitly this simple measure.

In 1951, Bohm [11] employed in the EPR argument a more convenient system of two spin-1/2 particles prepared in the entangled state of zero total spin J ,

$$|JM\rangle = |00\rangle = \frac{1}{\sqrt{2}} \left[\left| \frac{1}{2} \right\rangle_{\hat{\mathbf{n}}}^{(1)} \left| -\frac{1}{2} \right\rangle_{\hat{\mathbf{n}}}^{(2)} - \left| -\frac{1}{2} \right\rangle_{\hat{\mathbf{n}}}^{(1)} \left| \frac{1}{2} \right\rangle_{\hat{\mathbf{n}}}^{(2)} \right], \quad (7)$$

where $|\pm\frac{1}{2}\rangle_{\hat{\mathbf{n}}}^{(i)}$ is the spin state of particle i with the projection of its spin on the axis along an arbitrary unit vector $\hat{\mathbf{n}}$ equal to $\pm\frac{1}{2}$. The particles are assumed to be far away from each other; for example, they could be the s -wave products of the decay of a quasibound two-spin-1/2-particle system of zero total angular momentum. Since its introduction, the Bohm's version has been used almost exclusively in the literature as it has the advantage of using quantities that do not have continuous spectra and are conserved, thus freeing the discussion from non-essential aspects such as those connected with the non-normalizability of wavefunctions and the time evolution of the system, which may distract from the gist of the argument. Using the state (7), the EPR argument runs then as follows. The z -component s_{2z} of the spin of particle 2 is an element of reality because it can be predicted, without disturbing particle 2 in any way, by measuring the z -component s_{1z} of the spin of the distant particle 1: if such a measurement yields $s_{1z} = +\frac{1}{2}$ (or $-\frac{1}{2}$), it follows from the zero-total-spin state (7), with $\hat{\mathbf{n}} = \hat{\mathbf{z}}$, that s_{2z} must then equal $-\frac{1}{2}$ (or $+\frac{1}{2}$). But the x -component s_{2x} of the spin of particle 2 must be an element of reality, too, as one could predict it similarly by performing an alternative measurement of the x -component s_{1x} of the spin of particle 1 and using $\hat{\mathbf{n}} = \hat{\mathbf{x}}$ in (7). The quantum-mechanical description then must be incomplete as it does not allow a state in which both the z -component and x -component of the spin of a particle have definite values.

A comment on the role of the collapse of the wavefunction in the EPR argument seems now in order. While EPR talk about the collapse of the wavefunctions (1), (3) and (5) into single products of wavefunctions of subsystems 1 and 2 as a result of suitable measurements on subsystem 1, it can be seen easily that the collapse of the wavefunction does not need to be evoked explicitly in order to carry the argument through. The *joint* observables of the difference in the positions and of the sum of the momenta of two particles can be each measured by measuring the relevant observables on each of the two particles separately: the particles' individual positions for the first joint observable, and their individual momenta for the second one.⁷ But as the state (2) is an eigenstate of both the joint observables, the result of the measurement of the position or the momentum of particle 1, together with the knowledge of the eigenvalues of the joint observables in the state (2), i.e., the position difference x_0 and the

⁷However, such measurements of joint observables cannot be used to prepare the composite system in an entangled state like the one of Eq. (2); an experimental procedure that would prepare such an entangled state has been described by Bohr [10]. Moreover, such measurements, even when they are of the repeatable type, would disentangle a given entangled state of the composite system—and this is made use of in the argument of the rest of this section.

momentum sum 0, enables one to predict the value of the position or momentum of particle 2 already on the universally accepted meaning of an eigenstate and its eigenvalues—namely that, in such a state, a measurement of the observable (which is here of the joint type) pertaining to an eigenvalue of the state must yield with certainty that eigenvalue. Similarly, no collapse of the zero-total-spin state (7) is needed explicitly in the argument, only the fact that the total spin is zero and thus $s_{1z} + s_{2z} = s_{1x} + s_{2x} = 0$. Of course, it may be useful to couch the discussion in terms of suitable wavefunction collapses, but such collapses can be deduced here from the above mentioned meaning of an eigenstate and its eigenvalues with no need of postulating them as logically primary.⁸

3. “THE THIRD RELATED” ARGUMENT

The EPR argument employs measurements, performed or contemplated, on *only one of the two subsystems*, say particle 1, of the system. So far as particle 2 is concerned, its state is not directly measured, it is rather predicted on the result of a direct measurement on particle 1. However, the presentation of the EPR argument by Omnès employs measurements on *both* particles: in the Bohm’s version, a measurement of the z -component s_{1z} of particle 1 *and* a subsequent measurement (say at a time t) of the spin of particle 2 along another direction, say a measurement of the x -component s_{2x} . On the result of the measurement of s_{1z} one can predict the value of the z -component s_{2z} of the spin of particle 2 and thus, according to the EPR reality criterion, the value of s_{2z} of particle 2 is an element of physical reality, which, obviously, it will remain to be so until the time t of the direct measurement of s_{2x} . But the latter measurement assigns to particle 2 a definite value of s_{2x} , and so, at the time t , particle 2 must have definite values of both s_{2z} and s_{2x} , which is in direct conflict with the possibilities of the quantum-mechanical description. There is no fundamental reason, apart from that of convenience, for not performing the above measurements of s_{1z} and s_{2x} simultaneously; in fact, when the original example with position and momentum quantities is used in this version of the EPR argument, the position of one of the particles and the momentum of the other should be measured simultaneously to circumvent complications due to the time evolution of the system. Versions of the EPR argument that employ measurements on both the two subsystems belong to the class identified by

⁸Here it is noted that the “no-collapse measurements” proposed by Home and Whitaker [12], which are supposed to leave a system always in the state that the system had before measurement, can be ruled out using a similar argument. Indeed, if an entangled state of a composite system remained unchanged after the measurement of a suitable observable on one of its two subsystems, a subsequent measurement of the corresponding observable on the other subsystem would not necessarily yield a value that results in the eigenvalue of the relevant joint observable in the entangled state when the value is combined with the measurement result on the first subsystem.

Hooker and labeled by him as “the third related EPR argument.”⁹

The essential aspect in which “the third related” and original EPR arguments differ is that the former aims at establishing the possibility of simultaneous *determination* of two conjugate quantities, whereas in the latter it is argued for simultaneous *reality* (or *existence*) of such quantities. As is well known, one can find easily measurement procedures that assign simultaneously arbitrarily precise values to position and momentum, but they all have a retrospective character in the sense that they never prepare a state in which these values are the initial conditions.¹⁰ The uncertainty principle refers to the preparation of a state, which has testable consequences, rather than to a retrospective measurement with no predictive power. Einstein was well aware of that, and his failed pre-EPR attempts at disproving the uncertainty principle were aimed at finding experimental procedures that would lead to a simultaneous determination of two conjugate quantities in such a way that it would have a predictive power.¹¹ The EPR argument differs from the previous attempts of Einstein in not trying to show that one can determine simultaneously two conjugate quantities; EPR must have realized that simultaneous measurements of the position of one of the particles and the momentum of the other cannot lead to a determination with predictive power of the simultaneous position and momentum of any of the two particles. In fact, EPR steer carefully from what would be “the third related argument”: while they accept that the two conjugate quantities of particle 2 cannot be predicted at the same time because only one of the two alternative measurements on the correlated particle 1 can be performed actually, they do not counter this by proposing to perform the other measurement on particle 2 (this would be “the third related argument”). EPR argue “only” for simultaneous reality of the two conjugate quantities, using, with no need to perform in the end any measurements, their reality criterion in conjunction with their firm belief that the *real* state of a system cannot depend upon the process of local measurement performed on another system, well separated from the first system so that the measurement cannot disturb the latter. Bohr clearly understood that well, and aimed his reply to EPR accordingly at their concept of reality [10].

“The third related” EPR argument is thus not only a misrepresentation of the original EPR argument, but also lacks cogency: the simultaneous determination of two conjugate quantities it purports to establish has no predictive power; it cannot lead to a physical state in which the values of two conjugate

⁹A recent coincidence-measurement presentation of the EPR argument by Domingos *et al.* [13] thus falls in this class also; this paper has been commented on critically in Ref. [14].

¹⁰Heisenberg discussed such retrospective measurements on an example of the time-of-flight method of velocity measurement already in 1930 [15], and Bohr commented on such a method of velocity measurement even earlier, in his historic Como paper (Ref. [16], p. 66 in *Atomic Theory and the Description of Nature*). A more recent discussion of retrospective measurements can be found in Ref. [17].

¹¹Einstein’s famous clock-in-the-box thought experiment is perhaps the best example, see Ref. [18].

quantities are among its initial conditions and the knowledge of which would enable one to predict the future time development of the given system. To use the original EPR example with the conjugate quantities of position and momentum, a momentum measurement on particle 2 results in a loss of its spatial coordination with respect to the frame of reference, as demanded by the uncertainty principle, and the correlation it had with the position of particle 1, which could be used to determine its position, is thus lost irretrievably. Similarly, concerning now particle 1, its position measurement destroys irrecoverably the correlation it had with the momentum of particle 2 and which could be used to determine its momentum on the result of the momentum measurement on particle 2.¹² The prediction of the value of a quantity pertaining to a given particle on the result of a measurement of the similar quantity on the other particle, together with the simultaneous measurement of the conjugate quantity on the given particle, thus gives the simultaneous values of the two conjugate quantities of the given particle only in the retrospective sense.

Hooker in his excellent study of the Einstein–Bohr debate argues also against the cogency of “the third related” EPR argument,¹³ but for reasons that are criticized presently as flawed. Hooker asserts that in fact one cannot perform sufficiently accurately the simultaneous measurements of the position of particle 1 and the momentum of particle 2 because there is, as Bohr has shown [10,18], an uncontrollable transfer of momentum to the common coordinate frame as a result of the position measurement on particle 1. While it is certainly true that an accurate position measurement must lead to an uncontrollable transfer of momentum to the body that serves as a coordinate frame of reference, this circumstance presents no practical difficulty when the frame is sufficiently massive. Indeed, the momentum that a sufficiently massive frame absorbs in the position measurement of particle 1 results in the frame acquiring a negligible velocity, and thus the space-time coordination of any auxiliary body that is used in the determination of the momentum of particle 2 is not affected.¹⁴ As Bohr always stressed [18], proper space-time coordination uses a frame and clocks that can absorb the inevitable and uncontrollable transfers of momentum and

¹²Bohr stresses this point in his reply to EPR [10].

¹³See Ref. [2], Sec. 5 and pp. 223–224.

¹⁴A comment that has no direct bearing on the issues at hand seems nevertheless worth making here. To achieve a position determination of such an auxiliary body without affecting its momentum, Hooker proposes to use a parallel beam of light to illuminate a position pointer attached to the body, with a light detector placed behind the pointer, see Ref. [2], pp. 215–217. It can be seen easily, however, that also in such a procedure there will be an uncontrollable transfer of momentum to the body on account of the diffraction of light by the pointer’s edge. Such diffraction determines the ultimate accuracy of the position measurement and, as it involves an uncontrollable change in the direction of the incident photons, it leads to an uncontrollable transfer of momentum to the body along the direction in which its position is measured. Fortunately, the momentum uncertainty introduced by a position measurement does not affect the accuracy of the velocity determination by the time-of-flight method, in the context of which Hooker makes the above proposal, if the time interval between the two position measurements needed in such a method is sufficiently great [15].

energy without being affected in their role as a space-time framework. Thus, contrary to Hooker's assertion, the position of particle 1 and the momentum of particle 2 can be measured simultaneously and, in principle, with arbitrary accuracy. However, the assignment of simultaneous values to conjugate quantities of any of the two particles, to which such measurements lead in a procedure that follows "the third related" EPR argument, amounts simply to a retrospective measurement with no testable consequences.

4. CONCLUDING SUMMARY

A frequently given version of the EPR argument against the completeness of the quantum mechanical description, called by Hooker "the third related" EPR argument, was criticized as a misrepresentation of the original argument. The cogency of "the third related" argument itself was disputed on the grounds that it aims at establishing the possibility of a determination of simultaneous values of conjugate quantities, rather, as in the original EPR argument, at establishing their simultaneous reality, and that as such it amounts only to a scheme of a retrospective measurement.

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